Parabolized Navier-Stokes Analysis of Three-Dimensional Supersonic and Subsonic Jet Mixing Problems

S. M. Dash,* D. E. Wolf,† and N. Sinha†
Science Applications International Corporation, Princeton, New Jersey

Abstract

PARABOLIZED Navier-Stokes (PNS) models under development for the analysis of three-dimensional supersonic and subsonic propulsive jet mixing problems are described herein. The supersonic model, SCIP3D, is a direct extension of the 2D SCIPVIS PNS model of Dash and Wolf. It analyzes over- or underexpanded single and multiple rectangular jets employing a shock-capturing formulation. In the subsonic model, SPLIT3D, spatial marching capabilities are achieved via a pressure-splitting approximation and an extension of the two-dimensional noniterative cross-flow procedure of Dash and Sinha to three-dimensional flows. Both models contain $k\epsilon$ and kW two-equation turbulence model formulations and permit analyzing nonreacting multicomponent gas mixtures. Numerical studies are presented which demonstrate both codes' current capabilities.

Contents

The analysis of aircraft propulsive jets issuing from nonaxisymmetric nozzles requires the use of supersonic shock-capturing PNS methodology to treat the inner, shock-dominated region of the jet and subsonic pressure-split PNS methodology to treat the outer, vortex-entrainment dominated region for subsonic flight velocities. The ability to treat two-dimensional/axisymmetric jet problems using spatial marching PNS methodology has been demonstrated in recent publications by the authors. The SCIPVIS shock-capturing model of Dash and Wolf[1,2] treats the complex interactions between shock and turbulent mixing processes in supersonic jets.[3,4] The SPLITP model of Dash and Sinha[5,6] employs a pressure-splitting approach to treat subsonic jet mixing problems using a noniterative procedure[7] to treat pressure/cross flow velocity coupling in the cross flow plane. This paper describes preliminary extensions of SCIPVIS and SPLITP methodology to three-dimensional jet mixing problems as embodied in the SCIP3D and SPLIT3D models.

Three-Dimensional PNS Jet Equations

The conservation form of the three-dimensional PNS equations governing rectangular jet mixing are

$$\boxed{\frac{\partial \bar{E}}{\partial x}+\frac{\partial \bar{F}}{\partial y}+\frac{\partial \bar{G}}{\partial z}=\bar{V}_f} \tag{1}$$

where

$$\bar{f} = [1,U,V,W,H,\phi]^T$$

$$\bar{E}=\rho U[1,P/\rho U+U,V,W,H,\phi]^T$$

$$\bar{F}=\rho V[1,U,P/\rho V+V,W,H,\phi]^T$$

$$\bar{G}=\rho W[1,U,V,P/\rho W+W,H,\phi]^T$$

and the parabolized stress terms $\bar{V}_f$, are described in the full paper.

Supersonic PNS Model, SCIP3D

The mean flow and turbulence model equations are solved in mapped rectangular coordinates using a rotating version of the explicit MacCormack algorithm. The computational grid (Fig. 1) initially encompasses the viscous jet and the jet-induced interaction flowfield. For problems with quarter-plane symmetry, the initial spanwise and vertical boundary growth $[Y_u(x)$ and $Z_u(x)]$ is based on exact alignment of the viscous dividing streamlines in the symmetry planes with mapped coordinate lines. This minimizes numerical diffusion effects in the thin near-field shear layer region. Downstream, the computational boundary growth is adaptive and keyed to viscous edge gradients, transitioning from rectangular to square as the jet takes on a circular appearance. The captured plume induced bow shock ultimately exits the computational domain; downstream, one-wave flow relations are utilized permitting waves to exit the flow with no reflections back into the computational domain. Details are provided in the full paper.

A number of calculations are described in the full paper for square and rectangular (4/1 aspect ratio), balanced pressure and underexpanded ($P_J/P_E=2/1$) hot Mach 3 jets ($U_J=8820$ ft/s, $T_J=2000°$K) exhausting into a Mach 3 uniform external stream ($U_E=4410$ ft/s, $T_E=500°$K). Calculations are also described for the an infinite array of square underexpanded jets having the above conditions. Figure 2 exhibits a schematic of the multiple jet calculation and the predicted wave/shear layer structure and pressure contours in the $Z=0$ symmetry plane.

Subsonic PNS Model, SPLIT3D

In subsonic applications, the pressure field is "split" with the axial pressure gradient specified, and the cross-flow pressure variation determined via a coupled solution of the continuity and cross-flow momentum equations. The stepwise marching solution is comprised of a "parabolic" solution of Eq. (1) (achieved by treating the pressure gradient term in the axial momentum as a known source term, and using an upwind variant of the MacCormack predictor/corrector algorithm in the outer, entrainment dominated region of the jet) followed by a cross-flow solution. Previous cross-flow solutions have often exhibited convergence problems due to the strong coupling of the V and W velocity components in vortical-corner regions. In the approach developed here, the noniterative two-dimensional cross flow procedure of Dash and Sinha[7] is extended to the three-dimensional cross flow problem leading to a system of coupled equations for V and W which are solved concurrently

Presented as Paper 84-1525 at the AIAA 17th Fluid and Plasma Dynamics Conference, Snowmass, CO, June 25-27, 1984; received Aug. 28, 1984; synoptic submitted Nov. 20, 1985. Full paper available from AIAA Library, 555 W. 57th St., New York, NY 10019. Price: microfiche, $4.00; hard copy, $9.00. **Remittance must accompany order.**

*Technical Director, Propulsion Gas Dynamics Division. Member AIAA.

†Senior Research Scientist, Propulsion Gas Dynamics Division. Member AIAA.

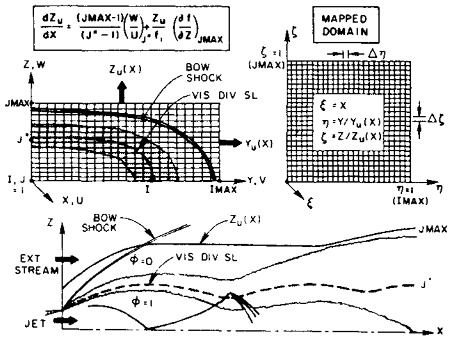

Fig. 1 Grid nomenclature, boundary growth formulation, and mapped computational domain utilized in SCIP3D model.

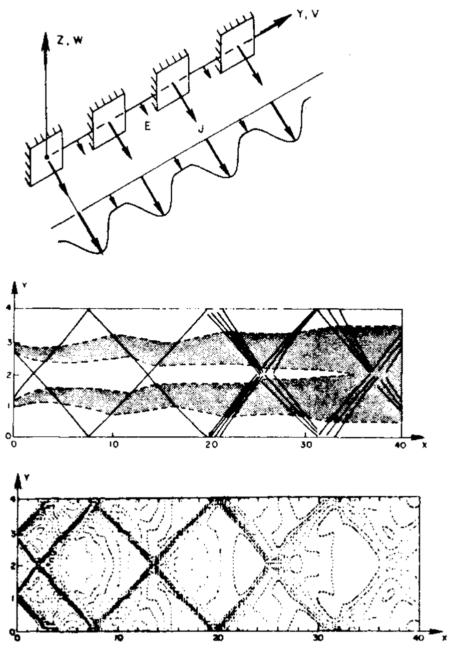

Fig. 2 Schematic of supersonic multiple square jet problem and predicted flow pattern and pressure contours in $Z=0$ symmetry plane.

with the pressure field lagged. The details are given in the full paper.

A series of SPLIT3D calculations were performed to analyze subsonic square and rectangular jets exhausting into a uniform subsonic external stream. All calculations simulated the parallel mixing of a Mach 0.26 jet exhausting into a Mach 0.13 external stream, with both the jet and external stream temperatures at 300°K. The jet exhaust was uniform and a step-like profile was employed to initiate the calculation at the nozzle exit plane. All calculations utilized a 21×21 grid. The axial pressure gradient $\partial P/\partial x$ was imposed to be zero in performing these calculations.

Figure 3 depicts species parameter (ϕ) contours in the two planes of symmetry for a rectangular jet problem with a 4/1 aspect ratio, showing the gradual transition towards circular jet behavior. Also depicted in Fig. 3 are near field cross flow contours (at $x=20$) of the species parameter exhibiting distinct kinks in the outer mixing region. The kinks in the ϕ contours are attributed to the breakdown of the initial vortex sheet surrounding the jet, with the subsequent formation of a pair of counter-rotating streamwise vortices. The vortices decay and the jet contours taken on a smooth elliptic cross-flow pattern by $X \sim 100$. The full paper exhibits the cross flow velocity vectors for the various cases performed, clearly exhibiting the characteristic features of the corner-induced vortex flow.

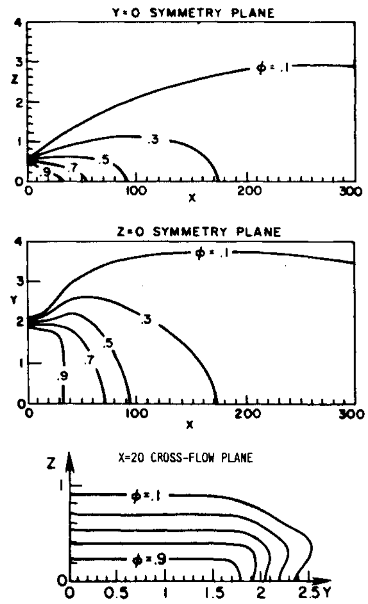

Fig. 3 Species contours in $Y=0$ and $Z=0$ symmetry planes and $X=20$ cross flow plane for subsonic rectangular jet problem.

Future Work

Future work is oriented towards the development of a unified model which analyzes supersonic jets exhausting into subsonic external streams. This model requires the development of an adaptive grid network which aligns the jet cross-flow sonic line with a cross-flow grid line to facilitate supersonic/subsonic coupling. The coupled problem has been solved for two-dimensional flows (Refs. 1-4) but the extension of this methodology to three-dimensional has many complexities which must be addressed (see Ref. 8).

Acknowledgments

This work was supported by NASA Langley Research Center, Hampton, Virginia under Contract NAS1-16535. The technical monitors were Dr. R. G. Wilmonth, Propulsion Aerodynamics Branch and Dr. J. M. Seiner, Aeroacoustics Branch.

References

[1]Dash, S. M. and Wolf, D. E., "Fully-Coupled Analysis of Jet Mixing Problems, Part I: Shock-Capturing Model, SCIPVIS," NASA CR-3761, Jan. 1984.

[2]Dash, S. M., Wolf, D. E., and Seiner, J. M., "Analysis of Turbulent Under-Expanded Jets, Part I: Parabolized Navier-Stokes Model, SCIPVIS," *AIAA Journal*, Vol. 23, April 1985, pp. 505-514.

[3]Dash, S. M. and Wolf, D. E., "Interactive Phenomena in Supersonic Jet Mixing Problems, Part I: Phenomenology and Numerical Modeling Techniques," *AIAA Journal*, Vol. 22, July 1984, pp. 905-913.

[4]Dash, S. M. and Wolf, D. E., "Interactive Phenomena in Supersonic Jet Mixing Problems, Part II: Numerical Studies," *AIAA Journal*, Vol. 22, Oct. 1984, pp. 1395-1404.

[5]Dash, S. M. and Sinha, N., "Fully-Coupled Analysis of Jet Mixing Problems, Part II: Pressure-Split Model, SPLITP," NASA CR in preparation.

[6]Dash, S. M., Beddini, R. A., Wolf, D. E., and Sinha, N., "Viscous/Inviscid Analysis of Curved Sub- or Supersonic Wall Jets," *AIAA Journal*, Vol. 23, Jan. 1985, pp. 12-13.

[7]Dash, S. M. and Sinha, N., "Noniterative Cross-Flow Integration for the Pressure-Split Analysis of Subsonic Mixing Layer Problems," *AIAA Journal*, Vol. 23, Jan. 1985, pp. 183-185.

[8]Dash, S. M., Wolf, D. E., Sinha, N., and Lee, S. H., "Progress in the Development of Parabolized Navier-Stokes (PNS) Methodology for Analyzing Propulsive Jet Mixing Problems," AIAA Paper 86-1115, May 1986.